# SEQUENCES AND SERIES

Precalculus

Chapter 10

1

- This Slideshow was developed to accompany the textbook
  - *Precalculus*
  - *By Richard Wright*
  - *https://www.andrews.edu/~rwright/Precalculus-RLW/Text/TOC.html*
- Some examples and diagrams are taken from the textbook.

Slides created by
Richard Wright, Andrews Academy
rwright@andrews.edu

3

# 10-01 SEQUENCES

In this section, you will:

- Write a sequence from a rule.
- Write an explicit rule for a sequence.
- Write a recursive rule for a sequence.
- Simplify factorial expressions.

# 10-01 SEQUENCES

- Sequence
  - List of numbers following a rule
  - 0, 3, 6, 9, 12 <− finite (ends)
  - 0, 3, 6, 9, 12, … <− infinite (doesn't end)

  - $n = 1, 2, 3, 4, 5, \ldots$ (term #) like $x$
  - $a_n = 0, 3, 6, 9, 12, \ldots$ (term value) like $y$

4

# 10-01 SEQUENCES

- Find the 1st 5 terms of $a_n = 5 + 2n(-1)^n$

5

$$a_1 = 5 + 2(1)(-1)^1 = 3$$
$$a_2 = 5 + 2(2)(-1)^2 = 9$$
$$a_3 = 5 + 2(3)(-1)^3 = -1$$
$$a_4 = 5 + 2(4)(-1)^4 = 13$$
$$a_5 = 5 + 2(5)(-1)^5 = -5$$

# 10-01 Sequences

- Write the rule for the $n^{th}$ term.
  1, 5, 9, 13, 17, …
- 2, -9, 28, -65, 126, …



$$a_n = 4n - 3$$

$$a_n = (-1)^{n+1}(n^3 + 1)$$

# 10-01 SEQUENCES

- Recursive Rules
  - Use the value of one term to find the next term.
  - $a_n$ means current term
  - $a_{n-1}$ means previous term
- Find the first 5 terms.
  - $a_1 = 6$
  - $a_n = a_{n-1} + 1$

7

$$a_1 = 6$$
$$a_2 = a_1 + 1 = 6 + 1 = 7$$
$$a_3 = a_2 + 1 = 7 + 1 = 8$$
$$a_4 = a_3 + 1 = 8 + 1 = 9$$
$$a_5 = a_4 + 1 = 9 + 1 = 10$$

# 10-01 Sequences

- Factorial (!)
  - Product of a whole number with all the whole numbers less than it through 1.
  - $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
  - $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
  - $0! = 1$
- Simplify

$$\frac{9!}{3!7!}$$

8

$$\frac{9!}{3!\,7!}$$

$$\frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$\frac{9 \cdot 8}{3 \cdot 2 \cdot 1}$$

$$\frac{72}{6}$$

$$12$$

# 10-01 Sequences

- Simplify

$\frac{(n+1)!}{n!}$

9

$$\frac{(n+1)!}{n!}$$
$$\frac{(n+1)(n)(n-1)(n-2)\cdots}{(n)(n-1)(n-2)\cdots}$$
$$n+1$$

# 10 10-02 SERIES

In this section, you will:

- Evaluate a summation.
- Write a series as a summation.

# 10-02 Series

- Series
  - Sum of a sequence
  - Sequence
    - 2, 4, 6, 8
  - Series
    - 2 + 4 + 6 + 8
- Summation Notation
  - (Sigma Notation)

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

11

i = index
1 = lower limit
n = upper limit

# 10-02 SERIES

- Find each sum

$$\sum_{i=1}^{4}(4i+1)$$

$$\sum_{k=2}^{5}(2+k^3)$$

12

$$\sum_{i=1}^{4}(4i+1) = (4(1)+1)+(4(2)+1)+(4(3)+1)+(4(4)+1)$$

$$= 5+9+13+17 = 44$$

$$\sum_{k=2}^{5}(2+k^3) = (2+2^3)+(2+3^3)+(2+4^3)+(2+5^3)$$

$$= 10+29+66+27 = 232$$

# 10-02 Series

$$\sum_{n=1}^{\infty} \frac{5}{10^n}$$



$$\sum_{n=1}^{\infty} \frac{5}{10^n}$$

Parital sums

$$\frac{5}{10^1} = 0.5$$

$$\frac{5}{10^1} + \frac{5}{10^2} = 0.5 + 0.05 = 0.55$$

$$\frac{5}{10^1} + \frac{5}{10^2} + \frac{5}{10^3} = 0.5 + 0.05 + 0.005 = 0.555$$

$$\frac{5}{10^1} + \frac{5}{10^2} + \frac{5}{10^3} + \frac{5}{10^4} = 0.5 + 0.05 + 0.005 + 0.0005 = 0.5555$$

Approaches

$$0.55555555555\ldots = \frac{5}{9}$$

# 10-02 Series

- Shortcut formulas

$$1 + 1 + 1 + 1 + \cdots = \sum_{i=1}^{n} 1 = n$$

$$1 + 2 + 3 + 4 + \cdots = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$1 + 4 + 9 + 16 + \cdots = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

# 10-02 SERIES

$$1 + 8 + 27 + 64 + \cdots = \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$1 + 16 + 81 + 256 + \cdots = \sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$1 + 32 + 243 + 1024 + \cdots = \sum_{i=1}^{n} i^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$$

15

# 10-02 Series

- Evaluate

$$\sum_{i=1}^{5}(3i^2 - 5i)$$

16

$$\sum_{i=1}^{5} 3i^2 - \sum_{i=1}^{5} 5i$$
$$= 3\left(\frac{n(n+1)(2n+1)}{6}\right) - 5\left(\frac{n(n+1)}{2}\right)$$
$$= 3\left(\frac{5(5+1)(2(5)+1)}{6}\right) - 5\left(\frac{5(5+1)}{2}\right)$$
$$= 90$$

# 10-03 ARITHMETIC SEQUENCES AND SERIES

17

In this section, you will:

- Write the explicit rule for an arithmetic sequence.
- Write the recursive rule for an arithmetic sequence.
- Evaluate the sum for an arithmetic series.

# 10-03 Arithmetic Sequences and Series

- Arithmetic
  - Common difference (d)
- 3, 7, 11, 15, 19, …
- Rule for the $n^{th}$ term
- $a_n = dn + c$
  - Where $c = a_1 - d$
- $a_n = a_1 + (n - 1)d$

18

# 10-03 Arithmetic Sequences and Series

- Find the rule for the $n^{th}$ term for 3, 7, 11, 15, 19, …

19

d = 7 – 3 = 4; 11 – 7 = 4, 15 – 11 = 4; 19 – 15 = 4

$a_1 = 3$

$$a_n = a_1 + (n-1)d$$
$$a_n = 3 + (n-1)4$$
$$a_n = 3 + 4n - 4$$
$$a_n = 4n - 1$$

# 10-03 ARITHMETIC SEQUENCES AND SERIES

- The 8th term of an arithmetic sequence is 25, and the 12th term is 41. Write the rule for the nth term.

20

$$a_n = a_1 + (n-1)d$$

Fill in $a_8 = 25$

$$25 = a_1 + (8-1)d$$
$$25 = a_1 + 7d$$

Fill in $a_{12} = 41$

$$41 = a_1 + (12-1)d$$
$$41 = a_1 + 11d$$

Solve system of eq.

$$-25 = -a_1 - 7d$$
$$\underline{41 = a_1 + 11d}$$
$$16 = \quad 4d$$
$$4 = d$$

$$25 = a_1 + 7(4)$$
$$a_1 = -3$$

$$a_n = -3 + (n-1)4$$
$$a_n = 4n - 7$$

# 10-03 ARITHMETIC SEQUENCES AND SERIES

- Recursive Rule for Arithmetic Sequences

- $a_1 = a_1$
- $a_n = a_{n-1} + d$

21

# 10-03 ARITHMETIC SEQUENCES AND SERIES

- Arithmetic Series
- $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$

- $1 \quad + \quad 3 + \quad 5 + \quad 7 + \quad 9$
- $\underline{19 + 17 + 15 + 13 + 11}$
- $20 + 20 + 20 + 20 + 20 = 5(20) = 100$

- $S_n = \frac{n}{2}(a_1 + a_n)$

22

# 10-03 Arithmetic Sequences and Series

- Find the sum of the integers 1 to 57.

23

$$1 + 2 + 3 + 4 + \cdots + 57$$

d = 1; $a_1 = 1$; $a_n = 57$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{57} = \frac{57}{2}(1 + 57)$$

$$S_{57} = \frac{57}{2}(58) = 1653$$

# 10-03 Arithmetic Sequences and Series

- Find the 50th partial sum of the arithmetic sequence -6, -2, 2, 6, …



$d = 4$; $a_1 = -6$

Find $n^{th}$ term to get $a_n$

$$a_n = a_1 + (n-1)d$$
$$a_n = -6 + (n-1)4$$
$$a_n = 4n - 10$$
$$a_{50} = 4(50) - 10 = 190$$

Find the sum

$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$S_{50} = \frac{50}{2}(-6 + 190) = 4600$$

# 10-03 Arithmetic Sequences and Series

- Evaluate

$$\sum_{i=1}^{100}(3i+2)$$



Linear (in form of $a_n = dn + c$) so arithmetic

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{100} = \frac{100}{2}\big((3(1)+2)+(3(100)+2)\big)$$

$$= 50(5+302)$$

$$= 15350$$

26

# 10-04 GEOMETRIC SEQUENCES AND SERIES

In this section, you will:

- Write the explicit rule for a geometric sequence.
- Write the recursive rule for a geometric sequence.
- Evaluate the sum for a geometric series.

# 10-04 Geometric Sequences and Series

- Geometric
  - Common ratio (r)
- 1, 3, 9, 27, 81, 243, …
- Rule for n<sup>th</sup> term
- $a_n = a_1 r^{n-1}$

- Find the rule for $6, -2, \frac{2}{3}, \ldots$

27

$$r = -\frac{2}{6} = -\frac{1}{3}$$
$$a_n = a_1 r^{n-1}$$
$$a_n = 6\left(-\frac{1}{3}\right)^{n-1}$$

# 10-04 Geometric Sequences and Series

- The 2nd term of a geometric sequence is -18, the 5th term is 2/3. Find the rule for the nth term.

28

$$a_n = a_1 r^{n-1}$$

Fill in $a_2 = -18$

$$-18 = a_1 r^{(2-1)}$$
$$-18 = a_1 r$$
$$a_1 = -\frac{18}{r}$$

Fill in $a_5 = \frac{2}{3}$

$$\frac{2}{3} = a_1 r^{5-1}$$
$$\frac{2}{3} = a_1 r^4$$

Substitute

$$\frac{2}{3} = -\frac{18}{r} r^4$$
$$\frac{2}{3} = -18r^3$$
$$-\frac{1}{27} = r^3$$

$$-\frac{1}{3} = r$$

Substitute

$$-\frac{18}{-\frac{1}{3}} = a_1$$

$$a_1 = 54$$

$$a_n = 54\left(-\frac{1}{3}\right)^{n-1}$$

# 10-04 Geometric Sequences and Series

- Geometric Series
- $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$
- $S_\infty = \frac{a_1}{1-r}$
  - Where $|r| < 1$

- Evaluate

$$\sum_{n=1}^{7} 2^{n-1}$$

29

$2^{n-1}$ is in the form $a_1 r^{n-1}$ where $a_1 = 1$ and $r = 2$

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

$$S_7 = 1\left(\frac{1-2^7}{1-2}\right) = 127$$

# 10-04 Geometric Sequences and Series

- Evaluate

  $5 + 0.5 + 0.05 + 0.005 + \cdots$

  $$\sum_{n=0}^{\infty} 5\left(\frac{1}{2}\right)^n$$

30

$a_1 = 5$ and $r = \frac{1}{10}$

$$S_\infty = \frac{a_1}{1-r}$$

$$= \frac{5}{1-\frac{1}{10}}$$

$$= \frac{5}{\frac{9}{10}} = 5\left(\frac{10}{9}\right) = \frac{50}{9}$$

The lower limit is not 1, so calculate n = 0

$$5\left(\frac{1}{2}\right)^0 + \sum_{n=1}^{\infty} 5\left(\frac{1}{2}\right)^n$$

The exponent is not n-1, so multiply and divide by ½

$$5 + \sum_{n=1}^{\infty} 5\left(\frac{1}{2}\right)\frac{\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$$

$$5+\sum_{n=1}^{\infty}\left(\frac{5}{2}\right)\left(\frac{1}{2}\right)^{n-1}$$

$a_1 = \frac{5}{2}$ and $r = \frac{1}{2}$

$$S_{\infty} = \frac{a_1}{1-r}$$

$$5+\left(\frac{\frac{5}{2}}{1-\frac{1}{2}}\right)$$

$$5+\left(\frac{\frac{5}{2}}{\frac{1}{2}}\right)$$

$$5+5=10$$

# 10-05 MATHEMATICAL INDUCTION

31

In this section, you will:

- Write a proof for a sum formula using mathematical induction.
- Prove other mathematical statements using mathematical induction.

# 10-05 Mathematical Induction

- Proofs for sum formulas
  - Show it works when $n = 1$
  - Show it works for $n + 1$

- Steps

1. Show it works for $n = 1$
2. Assume formula works for $n = k$
3. Show it works for $n = k + 1$
   - If proving sum formula use $S_{k+1} = S_k + a_{k+1}$

32

# 10-05 Mathematical Induction

- Prove $5 + 7 + 9 + 11 + 13 + \cdots + (3 + 2n) = n(n + 4)$



This is in the form $a_1 + a_2 + a_3 + \cdots + a_n = S_n$, so

$$a_n = 3 + 2n$$
$$S_n = n(n + 4)$$

1. $S_1 = 1(1 + 4) = 5 = a_1$
2. Assume $S_k = k(k + 4)$
3. $S_{k+1} = S_k + a_{k+1}$

$$(k + 1)\big((k + 1) + 4\big) = k(k + 4) + \big(3 + 2(k + 1)\big)$$
$$(k + 1)(k + 5) = k^2 + 4k + 3 + 2k + 2$$
$$k^2 + 6k + 5 = k^2 + 6k + 5$$

# 10-05 Mathematical Induction

- Prove $1(1-1)+2(2-1)+3(3-1)+\cdots+n(n-1)=\frac{n(n-1)(n+1)}{3}$



This is in the form $a_1+a_2+a_3+\cdots+a_n=S_n$, so

$$a_n = n(n-1)$$

$$S_n=\frac{n(n-1)(n+1)}{3}$$

1. $S_1=\frac{1(1-1)(1+1)}{3}=0=a_1$
2. Assume $S_k=\frac{k(k-1)(k+1)}{3}$
3. $S_{k+1}=S_k+a_{k+1}$

$$\frac{(k+1)\big((k+1)-1\big)\big((k+1)+1\big)}{3}=\frac{k(k-1)(k+1)}{3}+(k+1)\big((k+1)-1\big)$$

$$\frac{(k+1)(k)(k+2)}{3}=\frac{k^3-k}{3}+k^2+k$$

$$\frac{k^3+3k^2+2k}{3}=\frac{k^3-k}{3}+\frac{3k^2}{3}+\frac{3k}{3}$$

$$\frac{k^3+3k^2+2k}{3}=\frac{k^3+3k^2+2k}{3}$$

# 10-05 Mathematical Induction

- Prove $(n+1)! > 2^n$ where $n \geq 2$



1. $n = 2$: $(2+1)! > 2^2$
   - $3! > 4$
   - $6 > 4$
2. Assume $(k+1)! > 2^k$
3. Show $n = k+1$: $\big((k+1)+1\big)! > 2^{k+1}$
   - $(k+2)! > 2^{k+1}$
   - $(k+2)(k+1)! > 2^k \cdot 2$
   - $(k+2) > 2$ always and $(k+1)! > 2^k$ was true from step 2, so whole thing true.

# 10-05 Mathematical Induction

- Prove 4 is a factor of $5^n - 1$

36

1. $n = 1: 5^1 - 1 = 4$; 4 is a factor of 4
2. Assume 4 is a factor of $5^k - 1$
3. $n = k + 1$
   - $5^{k+1} - 1$
   - $5^{k+1} - 5^k + 5^k - 1$
   - $(5^k \cdot 5 - 5^k) + (5^k - 1)$
   - $5^k(5 - 1) + (5^k - 1)$
   - $4 \cdot 5^k + (5^k - 1)$
   - 4 is factor of both $4 \cdot 5^k$ and $(5^k - 1)$

# 10-06 BINOMIAL THEOREM

37

In this section, you will:

- Evaluate combinations.
- Expand binomial expressions.

# 10-06 Binomial Theorem

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $(x + y)^0$ | | | | | $1$ | | | | |
| $(x + y)^1$ | | | | $1x$ | | $1y$ | | | |
| $(x + y)^2$ | | | $1x^2$ | | $2xy$ | | $1y^2$ | | |
| $(x + y)^3$ | | $1x^3$ | | $3x^2y$ | | $3xy^2$ | | $1y^3$ | |
| $(x + y)^4$ | $1x^4$ | | $4x^3y$ | | $6x^2y^2$ | | $4xy^3$ | | $1y^4$ |

Properties

1. $n + 1$ terms
2. Powers of $x$ count down, $y$'s count up
3. Sum of exponents of each term = $n$
4. Coefficients are symmetrical

$_nC_r$

38

Rows are n starting at 0
Down to left diagonals are r starting at 0
nCr is the coefficient of the row and diagonal

# 10-06 Binomial Theorem

- Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$

- Where ${}_nC_r = \frac{n!}{(n-r)!r!}$

- Evaluate
- ${}_9C_2$
- $\binom{11}{4}$
- $\binom{8}{8}$
- $\binom{4}{2}$

39

On TI graphing calc: MATH → PRB | nCr

${}_9C_2$

Type 9 nCr 2 = 36

$$\binom{11}{4} = {}_{11}C_4 = 330$$

$$\binom{8}{8} = {}_8C_8 = 1$$

$$\binom{4}{2} = {}_4C_2 = 6$$

# 10-06 Binomial Theorem

- Expand $(x+2)^4$



a = x; b = 2; n = 4

$$\sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$

$${}_4C_0 x^4 2^0 + {}_4C_1 x^3 2^1 + {}_4C_2 x^2 2^2 + {}_4C_3 x^1 2^3 + {}_4C_4 x^0 2^4$$

$$1 \cdot x^4 \cdot 1 + 4 \cdot x^3 \cdot 2 + 6 \cdot x^2 \cdot 4 + 4 \cdot x \cdot 8 + 1 \cdot 1 \cdot 16$$

$$x^4 + 8x^3 + 24x^2 + 32x + 16$$

# 10-06 Binomial Theorem

- Expand $(3-x^2)^5$

41

a = 3; b = $-x^2$; n = 5

$$\sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$

$${}_5C_0 3^5(-x^2)^0 + {}_5C_1 3^4(-x^2)^1 + {}_5C_2 3^3(-x^2)^2 + {}_5C_3 3^2(-x^2)^3 + {}_5C_4 3^1(-x^2)^4 + {}_5C_5 3^0(-x^2)^5$$

$$1 \cdot 243 \cdot 1 + 5 \cdot 81 \cdot (-x^2) + 10 \cdot 27 \cdot x^4 + 10 \cdot 9 \cdot (-x^6) + 5 \cdot 3 \cdot x^8 + 1 \cdot 1 \cdot (-x^{10})$$

$$243 - 405x^2 + 270x^4 - 90x^6 + 15x^8 - x^{10}$$

# 10-06 BINOMIAL THEOREM

- Find the coefficient of the term $a^4b^7$ in $(2a-3b)^{11}$

42

a = 2a; b = -3b; n = 11

$$\sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$

Compare this to $a^4b^7$ to see that r = 7

$${}_{11}C_7(2a)^{11-7}(-3b)^7$$
$$330(16a^4)(-2187b^7)$$
$$-11{,}547{,}360a^4b^7$$

-11,547,360

43

# 10-07 COUNTING PRINCIPLES

In this section, you will:

- Apply the fundamental counting principle
- Calculate permutations
- Calculate combinations

# 10-07 Counting Principles

- Fundamental Counting Principle
  - If events $E_1$ and $E_2$ occur in $m_1$ and $m_2$ ways, the number of ways both events can occur is $m_1 \cdot m_2$.

- A lock will open with the right choice of 3 numbers. How many different sets of 3 numbers can you choose if each number is from 1 to 30 inclusive? (a) with repetition (b) without repetition

44

a. $30 \cdot 30 \cdot 30 = 27000$
b. $30 \cdot 29 \cdot 28 = 24360$

# 10-07 Counting Principles

- How many license plates can be made if each is 2 letters follow by 4-digits? (a) with repetition (b) without repetition

45

a. $26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 6{,}760{,}000$
b. $26 \cdot 25 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 3{,}276{,}000$

# 10-07 COUNTING PRINCIPLES

- Permutation
  - Number of ways to **order** *n* objects taken *r* at a time
  - ${}_nP_r = \frac{n!}{(n-r)!}$

- How many ways can 8 children line up in a row?

46

n = 8; r = 8

$${}_8P_8 = \frac{8!}{(8-8)!} = 8! = 40{,}320$$

# 10-07 Counting Principles

- A club has 24 members, how many ways can 5 officers be selected?

47

Choose from 24 → n=24
Actually choosing 5 → r=5

$$_{24}P_5 = \frac{24!}{(24-5)!} = \frac{24\cdot 23\cdot 22\cdot 21\cdot 20\cdot 19\cdot 18\cdot 17\cdot 16\cdots}{19\cdot 18\cdot 17\cdot 16\cdots}$$
$$= 24\cdot 23\cdot 22\cdot 21\cdot 20$$
$$= 5{,}100{,}480$$

# 10-07 COUNTING PRINCIPLES

- Distinguishable Permutations
  - What is some objects are exactly the same?
  - ABB
    - BAB and BAB are the same eventhough the B's were switched
  - We want the orders that look different (choosing all the objects)

$$\frac{n!}{q_1! \cdot q_2! \cdot q_3! \cdots}$$

    - Where $n$ = number of objects; $q$ = how many times each is repeated

48

# 10-07 Counting Principles

- How many distinguishable ways to order the letters in BANANA?

49

n=6; q1=3 (A); q2=2 (N)

$$\frac{6!}{3! \cdot 2!} = 60$$

# 10-07 COUNTING PRINCIPLES

- Combinations
  - Grouping of objects **without order**
  - ABC is the same as BAC

$$ {}_nC_r = \frac{n!}{(n-r)!\,r!} $$

- There are 31 students. How many different groups of 4 can be made?

50

$$ {}_{31}C_4 = 31{,}465 $$

# 10-07 COUNTING PRINCIPLES

- You are forming a 10-person committee from 9 women and 12 men. How many different committees if 5 women and 5 men?

51

Order is not important → combination

Combination of women × combination of men

$$_{9}C_{5} \cdot {}_{12}C_{5} = 99{,}792$$

52

# 10-08 PROBABILITY

In this section, you will:

- Calculate simple probability
- Calculate the probability of compound events
- Calculate the probability of multiple events
- Calculate the complement of an event

# 10-08 PROBABILITY

- Probability
  - Number from 0 to 1 indicating how likely something is to happen.
  - 0 = never happens
  - 1 = always happens

$$P(A) = \frac{favorable\ outcomes}{total\ outcomes}$$

# 10-08 PROBABILITY

- A box contains 3 red marbles, 5 black marbles, and 2 yellow marbles. If a marble is selected at random, what is the probability of choosing yellow?

54

$$P(Y) = \frac{yellow}{total} = \frac{2}{10} = \frac{1}{5}$$

# 10-08 PROBABILITY

- 2 dice are rolled, what is the probability that the sum is 8?

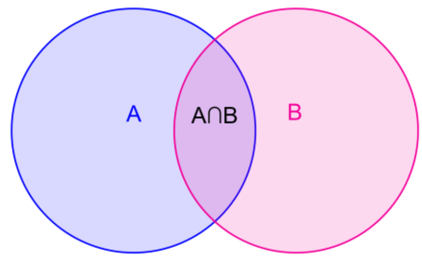
| | | | | | |
|---|---|---|---|---|---|
| 1 1 | 2 1 | 3 1 | 4 1 | 5 1 | 6 1 |
| 1 2 | 2 2 | 3 2 | 4 2 | 5 2 | 6 2 |
| 1 3 | 2 3 | 3 3 | 4 3 | 5 3 | 6 3 |
| 1 4 | 2 4 | 3 4 | 4 4 | 5 4 | 6 4 |
| 1 5 | 2 5 | 3 5 | 4 5 | 5 5 | 6 5 |
| 1 6 | 2 6 | 3 6 | 4 6 | 5 6 | 6 6 |

55

$$P(8) = \frac{8's}{total} = \frac{5}{36}$$

# 10-08 Probability

- Compound Events
  - 1 event with 2 accepted outcomes
  - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  - If $P(A \cap B) = 0$, then called mutually exclusive




56

# 10-08 PROBABILITY

- You draw one card from a standard 52-card deck. What is the probability of drawing a heart or red?

57

$$P(heart \cup red) = P(heart) + P(red) - P(heart \cap red)$$
$$= \frac{13}{52} + \frac{26}{52} - \frac{13}{52} = \frac{26}{52} = \frac{1}{2}$$

# 10-08 PROBABILITY

- Multiple Events
  - Two events with 2 outcomes
- Independent – Event A does not affect event B
  - $P(A \text{ and } B) = P(A) \cdot P(B)$
- Dependent – Event A does affect event B
  - $P(A \text{ and } B) = P(A) \cdot P(B|A)$
    - Where $P(B|A)$ is the probability that B occurs given that A already occured

58

# 10-08 PROBABILITY

- You draw 2 cards from a standard 52-card deck. What is the probability you draw a heart and a red? (a) with replacement (b) without replacement

59

a. $P(heart\ and\ red) = P(heart) \cdot P(red) = \frac{13}{52} \cdot \frac{56}{52} = \frac{1}{8} = 0.125$

b. $P(heart\ and\ red) = P(heart) \cdot P(red|heart) = \frac{13}{52} \cdot \frac{25}{52} = 0.123$

# 10-08 PROBABILITY

- Complement
  - Opposite
  - $P(\overline{A}) = 1 - P(A)$
  - At least one $P(n \geq 1)$ is easier with the complement never $P(0)$

60